Sample Standard Deviation

Figure 10B.3
S-Chart for C1

Questions and Problems

1. For the following results from samples drawn from normal populations, what are the best estimates for the mean, the variance, the standard deviation, and the standard deviation of the mean?

   a. $n = 9$, $\Sigma X_i = 36$, $\Sigma(X_i - \overline{X})^2 = 288$
   b. $n = 16$, $\Sigma X_i = 64$, $\Sigma(X_i - \overline{X})^2 = 180$
   c. $n = 25$, $\Sigma X_i = 500$, $\Sigma X_i^2 = 12{,}400$

2. For each of the following samples drawn from normal populations, find the best estimates for μ, σ^2, σ, and the standard deviation of $\overline{X}$.

   a. 4, 10, 2, 8, 4, 14, 12, 8, 10
   b. -4, 2, -6, 0, 6, 2, 4, 0, -4
   c. 6, 15, 13, 21, 10, 17, 12

3. Find the value of $z_{\alpha/2}$ for the following values of α.

   a. $\alpha = .01$ **b.** $\alpha = .03$ **c.** $\alpha = .002$

4. A stockbroker has taken a random sample of 4 stocks from a large population of low-priced stocks. Stock prices for this population are normally distributed. The sample prices of the 4 stocks are $5, $12, $17, and $10.

   a. Calculate a point estimate of the population mean.
   b. Calculate a point estimate of the population variance. What is your estimate for a population standard deviation?
   c. Calculate a point estimate of the proportion of stocks in this population that are priced at $10 or more.

5. A 90 percent confidence interval for the population mean time (in minutes) needed to finish a certain assembly process is $90 < \mu_x < 130$.

   a. Sketch this interval, indicating the margin for sampling error.

6. Assuming you have samples from normal populations with known variance, find
   a. The degree of confidence used if $n = 16$, $\sigma = 8$, and the total width of a confidence interval for the mean is 3.29 units.
   b. The sample size when $\sigma^2 = 100$ and the 95 percent confidence interval for the mean is from 17.2 to 22.8.
   c. The known variance when $n = 100$ and the 98 percent confidence interval for the mean is 28.26 units in width.

7. a. Find the value of t such that the probability of a larger value is .005 when the degrees of freedom is very large.
   b. Find the value of t such that the probability of a smaller value is .975 when the degrees of freedom is very large (infinite).
   c. Are the t values essentially the same as corresponding Z values when the value of the degrees of freedom is very large?

8. A poll reported that 48 percent of probable voters seem determined to vote against the President. Assume that this sample was based on a random selection of 789 probable voters. Construct a 99 percent confidence interval for the probable voters who seem determined to vote against the President.

9. A survey of low-income families in New Jersey was designed to determine the average heating costs for a family of 4 during January and February. Heating costs are known to have a standard deviation of $25.75. The economists conducting the study wish to construct a 95 percent confidence interval with a margin for sampling error of no more than $3.95. Find the appropriate sample size.

10. A company has just installed a new automatic milling machine. The time it takes the machine to mill a particular part is recorded for a sample of 9 observations. The mean time is found to be $\bar{X} = 8.50$ seconds, and $S^2 = .0064$. Find a 90 percent confidence interval for the unknown mean time for milling this part.

11. A survey indicated that companies with fewer than 1,000 employees are expected to increase their spending by 20.4 percent. Form a 99 percent confidence interval for the unknown mean increase, assuming that the sample standard deviation is 6.8 percent and the sample size is 346.

12. A study conducted in 1984 reported that the median pay in the United States was $18,700. What difficulties do you see in using this type of study for assessing incomes? Would you be willing to use $18,700 as a point estimate of the central location of U.S. incomes?

13. What is a point estimate? What is a point estimator? What is point estimation? How are these concepts related to the concepts of sampling that we discussed in Chapter 9?

14. What is an unbiased estimator? What is an efficient estimator? What is a consistent estimator? Why are these concepts important?

15. Briefly explain why we sometimes construct confidence intervals for the population mean.

16. Explain what happens to the size of the confidence interval when
    a. The standard deviation increases.
    b. The standard deviation decreases.
    c. The probability content $(1 - \alpha)$ increases from 95 percent to 99 percent.
    d. The sample size increases from 100 to 1,000.

17. Labor economists at the Department of Labor say they have 95 percent confidence that factory workers' earnings will lie between $22,000 and $61,000. Explain what this means.

18. A real estate agent in Connecticut is interested in the mean home price in the state. A random sample of 50 homes shows a mean home price of $175,622 and a sample standard deviation of $37,221. Construct a 95 percent confidence interval for the mean home price.

19. Reconstruct the confidence interval for the mean home prices given in question 18, but this time construct a 99 percent confidence interval. What happens to the size of the confidence interval?

20. Again, use the information given in question 18. This time assume that 100 homes are randomly sampled instead of 50. Construct a 95 percent confidence interval for the mean home price. What happens to the size of the confidence interval?

21. Again, use the information given in question 18. This time, assume that the sample standard deviation is $28,000. Construct a 95 percent confidence interval for the mean home price. What happens to the size of the confidence interval?

22. An auditor randomly samples 75 accounts receivable of a company and finds a sample mean of $128 with a sample standard deviation of $27. Construct a 90 percent confidence interval for the mean accounts receivable.

23. A random sample of 300 residents of a town shows that 55 percent believe the mayor is doing a good job. Construct a 95 percent confidence interval for the proportion of all residents who believe the mayor is doing a good job.

24. A random sample of 200 students at Academic University finds the sample mean grade point average to be 3.10 with a standard deviation of .80. Construct a 99 percent confidence interval for the mean grade point average.

25. An insurance company is interested in the average claim on its auto insurance policies. It believes the claims are normally distributed. Using the last 37 claims, it finds the mean claim to be $1,270 with a standard deviation of $421. Construct a 95 percent confidence interval for the mean claim on all policies.

26. A random sample of the luggage of 30 passengers of Fly Me Airlines finds that the mean weight of the luggage is 47 pounds with a standard deviation of 8 pounds. Construct a 90 percent confidence interval for the mean weight of Fly Me Airlines luggage.

27. A bank manager finds from reviewing her records that the amount of money deposited on Saturday morning is normally distributed with a standard deviation of $150. A random sample of 7 customers reveals the following amounts deposited on Saturday morning.

$825 $972 $311 $1,212 $150 $1,800 $725

  a. Find a 95 percent confidence interval for the mean amount of deposits by using the MINITAB program.
  b. Find a 90 percent confidence interval for the mean amount of deposits by using MINITAB

again. Compare your answer to the confidence interval you computed in part (a). Which is larger?

28. Redo question 27, parts (a) and (b), this time assuming the population standard deviation is unknown. Use MINITAB.

29. A quality control engineer believes that the life of light bulbs for his company is normally distributed with a standard deviation of 100 hours. A random sample of 10 light bulbs gives the following information on the life of the light bulbs.

  1,000 hours; 1,200 hours; 600 hours; 400 hours; 900 hours; 500 hours; 1,520 hours; 1,800 hours; 300 hours; 525 hours

  a. Find a 90 percent confidence interval for the mean life of the light bulbs.
  b. Suppose the standard deviation is not known. Construct a 90 percent confidence interval for the mean life of the light bulbs.

30. Managers at the Smooth Ride Car Rental Company are interested in the mean number of miles that people drive per day. From past experience, they know that the standard deviation is 75 miles. A random sample of 6 car rentals shows that the people drove the following numbers of miles: 152, 222, 300, 84, 90, 122. Construct a 99 percent confidence interval for the mean number of miles driven.

31. A credit manager at the Bargain Basement Department Store is interested in the proportion of customers who pay their credit card balances in full each month. A random sample of 200 customers indicates that 95 paid their balance in full each month. Construct a 99 percent confidence interval for the proportion of customers who pay their balances in full each month.

32. Construct point estimates for the following situations.
  a. A labor union randomly samples 75 of its members and finds that 40 favor the new contract. Estimate the proportion of all workers who favor the new contract.
  b. An economics professor randomly samples 100 students in her class and finds that 70 do not know the meaning of *elasticity*. Estimate the proportion of all students in her class who cannot define this term.

33. An auditor randomly samples 50 accounts payable of a company and finds a sample mean of $1,100 with a sample standard deviation of $287. Construct a 90 percent confidence interval for the mean accounts payable.

34. A random sample of 250 residents of a town shows that 55 percent favor a bond issue to finance new school construction. Construct a 99 percent confidence interval for the proportion of all residents who favor the bond issue.

35. A random sample of 500 students at Average College finds the sample mean combined-SAT score to be 1,050 with a standard deviation of 120. Construct a 90 percent confidence interval for the mean SAT score.

36. Reviewing his records, a grocery store manager finds that the amount of money spent shopping on Friday evenings is normally distributed with a standard deviation of $22. A random sample of 5 customers reveals the following amounts spent shopping on Friday night: $125, $72, $15, $88, $96.

  a. Find a 95 percent confidence interval for the mean amount of money spent shopping.

  b. Find a 90 percent confidence interval for the mean amount of money spent shopping. Compare your answer to the confidence interval you computed in part (a). Which is larger?

37. A random sample of 75 observations from a population yielded the following summary statistics:

$$\Sigma x = 1,270 \qquad \Sigma x^2 = 21,520$$

Construct a 95 percent confidence interval for the population mean μ.

38. A random sample of 100 observations from a population yielded the following summary statistics:

$$\Sigma x_i = 375 \qquad \Sigma(x_i - \bar{x})^2 = 972$$

Construct a 99 percent confidence interval for the population mean μ.

39. Suppose a random sample of 40 professional golfers is taken and the mean scoring average of the sample is found to be 72.8 strokes per round with a standard deviation of 1.2 strokes per round. Construct a 99 percent confidence interval for the population's mean strokes per round.

40. Suppose a random sample of 10 professional golfers is taken and the mean scoring average of the sample is found to be 71.8 strokes per round with a standard deviation of 1.3 strokes per round. Construct a 90 percent confidence interval for the population mean strokes per round.

41. Reconsider the information given in question 40. Suppose now that the population standard deviation is known to be 1.3 strokes per round. Construct a 90 percent confidence interval for the population mean strokes per round. Compare your answer to your answer in question 40. Why are they different?

42. Suppose you construct a 95 percent confidence interval for the mean of an infinite population. Will the interval always be narrower when σ is known than when σ is unknown?

43. A random sample of 75 observations reveals that the sample mean is 20. You know that the population standard deviation is 5. Construct a 90% confidence interval for the population mean.

44. In a national survey, 200 cola drinkers were asked to compare Yum Yum Cola to Yuk Yuk Cola. Of the 200 people sampled, 120 preferred Yum Yum. Construct a 95 percent confidence interval for the actual proportion of consumers who prefer Yum Yum Cola.

45. The 80 members of a random sample of graduates of Mary's Typing School indicate that their mean salary is $22,500 with a standard deviation of $3,100. Construct a 99 percent confidence interval for the true mean salary.

46. Suppose a random sample of 30 college students reveals that the mean amount of money spent on textbooks each semester is $145 with a standard deviation of $25. Construct a 90 percent confidence interval for the mean amount of money that students spend on textbooks each semester.

47. The Better Health Cereal Company produces Healthy Oats cereal. A sample of 100 boxes of this cereal indicates that the mean weight of a box of cereal is 24 ounces with a standard deviation of 1 ounce. Construct a 99 percent confidence interval for the population's mean weight.

48. The Better Health Cereal Company produces Healthy Oats cereal. A sample of 15 boxes of this cereal indicates that the mean weight of a box of